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(1) Do not merely verify but deduce the equation and find x_1. (2) Deduce the x-equation (x_1, x_2, x_3, x_4 the roots) such that its greatest root x_1 gives $\cos(90°/9) = \cos 10° = \frac{1}{2}\sqrt{x_1}$. (3) Of what angles are $\frac{1}{2}\sqrt{x_1}, \cdots \frac{1}{2}\sqrt{x_4}$ in (2) the cosines? Develop a method of writing out at once $\cos nv$ in terms of powers of $\cos v$ if these are given for $(n-1)v$ and $(n-2)v$. The same for $\sin nv$. (4) Use the results of (2) and (3) to find the number of degrees in a radian. Hence, find π from radian instead of radian from π as is usual.

Solution by J. B. Reynolds, Lehigh University.

From the formula

$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y),$$

letting $x = nv$ and $y = (n-2)v$, we have[1]

$$\cos nv = 2 \cos v \cos (n-1)v - \cos (n-2)v.$$

So also $\sin nv = 2 \cos v \sin (n-1)v - \sin (n-2)v$.
Putting $n = 2, 3, \cdots 9$, we readily obtain

$$\cos 7v = 64 \cos^7 v - 112 \cos^5 v + 56 \cos^3 v - 7 \cos v,$$

and

$$\cos 9v = 256 \cos^9 v - 576 \cos^7 v + 432 \cos^5 v - 120 \cos^3 v + 9 \cos v.$$

Replacing in these $\cos v = \frac{1}{2}\sqrt{x}$, we get

$$\cos 7v = \frac{x^{1/2}}{2}(x^3 - 7x^2 + 14x - 7), \tag{1}$$

$$\cos 9v = \frac{x^{1/2}}{2}(x^4 - 9x^3 + 27x^2 - 30x + 9). \tag{2}$$

If in (1), we let $\cos 7v = 0$ or $7v = (n + \frac{1}{2})\pi$, we have $v = \pi/14 = 90°/7$ for the greatest root $4 \cos^2 v$, of

$$x^3 - 7x^2 + 14x - 7 = 0. \tag{3}$$

Again, if we let $\cos 9v = 0$ or $9v = (n + \frac{1}{2})\pi$, we have $v = (2n + 1)\pi/18$ for the roots of

$$x^{1/2}(x^4 - 9x^3 + 27x^2 - 30x + 9) = 0; \tag{4}$$

and if we call them x_1, x_2, x_3, x_4, 0, for $n = 0, 1, 2, 3, 4$, then

$$x_1 = 4 \cos^2 \pi/18, \quad x_2 = 4 \cos^2 \pi/6, \quad x_3 = \cos^2 5\pi/18, \quad x_4 = 4 \cos^2 7\pi/18, \quad x_5 = 4 \cos^2 \pi/2 = 0.$$

Since $4 \cos^2 \pi/6 = 3$, we may factor (4) into

$$x^{1/2}(x - 3)(x^3 - 6x^2 + 9x - 3) = 0. \tag{5}$$

We may solve (3) and (5) by Horner's method to get any of the roots to the desired degree of accuracy, since the roots are all real. In this manner, we find from (5) $x_1 = 3.87938$, $x_3 = 1.65271$, and since $x_1 + x_3 + x_4 = 6$, $x_4 = .46791$, $x_2 = 3$. Now we have $\cos 70° = \frac{1}{2}\sqrt{x_4}$ or

$$70° = \cos^{-1}(\tfrac{1}{2}\sqrt{x_4}) = 90° - \sin^{-1}(\tfrac{1}{2}\sqrt{x_4}),$$

giving

$$20° \text{ in radians} = \sin^{-1}\left(\frac{{x_4}^{1/2}}{2}\right) = \frac{{x_4}^{1/2}}{2}\left\{1 + \frac{x_4}{24} + \frac{3{x_4}^2}{640} + \frac{5{x_4}^3}{64 \cdot 112} + \cdots\right\}$$

$$= .349058 \text{ radians};$$

whence, 1 radian $= 57.297°$ or $\pi/9 = .349058$, giving $\pi = 3.14152$.

2830 [1920, 227]. Proposed by WILLIAM HOOVER, Columbus, Ohio.

An elastic string connects a pair of opposite vertices of a square whose sides are four equal rods freely jointed, each of length $2a$. The system is suspended vertically from a vertex attached to the string and is at rest. If the string be cut, and θ is the acute angle any side makes with the vertical at any moment during the motion, determine the angular velocity of any rod.

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[1] Compare Note 3, in this Monthly, *1921*, 38–39.

SOLUTION BY THE PROPOSER.

Change the word square in the problem to rhombus. Take the horizontal line through the fixed vertex for the x-coordinate axis, and the vertical diagonal of the rhombus formed of the rods for the y-axis. Let x_1, y_1 be the numerical values of the coordinates of each of the centers of the upper pair of rods; x_2, y_2 of each of the lower pair; m the mass of each rod; and α the value of θ upon the cutting of the string.

We have $x_1 = a \sin \theta$, $y_1 = a \cos \theta$; $x_2 = a \sin \theta$, $y_2 = 2a \cos \theta + a \cos \theta = 3a \cos \theta$.

The energy equation for the motion is

$$2\cdot\tfrac{1}{2}m\left(\dot{x}_1^2 + \dot{y}_1^2 + \frac{a^2}{3}\dot{\theta}^2\right) + 2\cdot\tfrac{1}{2}m\left(\dot{x}_2^2 + \dot{y}_2^2 + \frac{a^2}{3}\dot{\theta}^2\right) = 2mgy_1 + 2mgy_2 + C;$$

but $\dot{x}_1 = \dot{x}_2 = a \cos \theta\cdot\dot{\theta}$, $\dot{y}_1 = -a \sin \theta\cdot\dot{\theta}$, $\dot{y}_2 = -3a \sin \theta\cdot\dot{\theta}$; then

$$m\frac{4a^2}{3}\dot{\theta}^2 + ma^2\left(\frac{4}{3} + 8\sin^2\theta\right)\dot{\theta}^2 = 8mag\cos\theta + C.$$

But $\theta = \alpha$ when $\dot{\theta} = 0$; then $C = -8mag \cos \alpha$, and the required angular velocity is given by

$$a(1 + 3\sin^2\theta)\dot{\theta}^2 = 3g(\cos\theta - \cos\alpha) \qquad \text{or,} \qquad \dot{\theta} = \sqrt{\frac{3g(\cos\theta - \cos\alpha)}{a(1 + 3\sin^2\theta)}}.$$

Also solved by J. B. REYNOLDS and F. L. WILMER.

2836 [1920, 273]. Proposed by W. V. N. GARRETSON, Rutgers College.

A ladder 40 feet long rests with one end on the ground against the foot of a building and the other end against the side of a second building directly across the street from the first. A second ladder 25 feet long inclines in a similar manner from the foot of the second building against the side of the first building, the two ladders crossing at a point 15 feet above the ground. How wide is the street?

SOLUTION BY H. S. UHLER, Yale University.

Let x and y denote respectively the width of the street and the distance from the foot of the first building to the lowest point of the 15 ft. vertical.

From similar right triangles, we obtain immediately

$$\frac{15}{\sqrt{1600 - x^2}} = \frac{y}{x}$$

and

$$\frac{15}{\sqrt{625 - x^2}} = \frac{x - y}{x} = 1 - \frac{y}{x}.$$

Therefore, by addition,

$$\frac{15}{\sqrt{1600 - x^2}} + \frac{15}{\sqrt{625 - x^2}} - 1 = 0. \tag{1}$$

The solution of equation (1) may be effected advantageously in the following manner. Let the second fraction be represented by $1/u$ so that

$$x = 5\sqrt{25 - 9u^2}, \tag{2}$$

and equation (1) transforms into

$$\frac{\sqrt{3}}{\sqrt{13 + 3u^2}} = \frac{u - 1}{u},$$

or

$$3u^4 - 6u^3 + 13u^2 - 26u + 13 = 0.$$

Solving the last equation by Horner's method of approximation we obtain $u = 1.612347811$. The required result is now found by substituting in relation (2). It is $x = 6.33050319$ ft.

Also solved by T. M. BLAKSLEE, E. B. ESCOTT, LAURA GUGGENBUHL, W. W. JOHNSON, G. A. KNAPP, R. M. MATHEWS, H. L. OLSON, ARTHUR PELLETIER, J. B. REYNOLDS, and C. C. WYLIE.